# Working Compositions for Correct Execution of Robot Task Specifications

*D. E. Koditschek*

*ESE Department, University of Pennsylvania*

*Philadelphia, PA 19104*

***The research discussed in this talk has been supported by***

OSD/R&E(BRO)/ONR: Vannevar Bush Fellowship (N00014-16-1-2817)
AFOSR: FA9550-19-1-0265; AFRL: FA865015D1845 sub 669737-1D
ARO: W911NF-17-1-0229; ARO: SLICE MURI W911NF1810327
NSF: LLDR (INSPIRE-1514882); CoRoGeoFR (NRI-II-1734355)

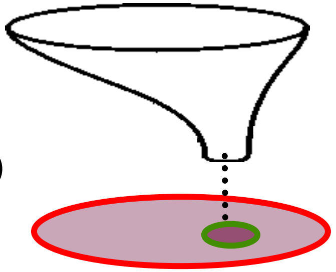
Penn Engineering | GRASP Laboratory
General Robotics, Automation, Sensing & Perception Lab

# Robotics: Work ⇒ Dynamics ⇒ Basins ⇒ Composition

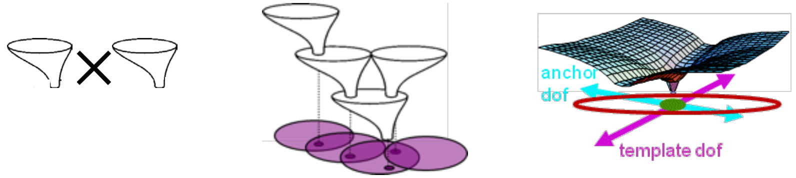
[Kod, Annu. Rev. CRAS’21]

*Task: prescribe and execute an exchange of energy with the environment* ⇔ ***attractor basin***

- Tasks: Architecture ↔ Environment
- Closed Loop Dynamical System
  - Newton (1687) – *dynamics*: $F = ma$
  - Lord Kelvin (1888) – *energy*: $d/dt\ E = P - D$
  - Lyapunov (1892) – *stability*: $d/dt\ E = \nabla E \cdot f(x)$
  - Poincare’ (1895) – *topology*: $\alpha\beta\alpha^{-1}\beta^{-1} \neq \iota$
  - Conley (1978) – *chain recurrence*: $\mathcal{R}(\phi^t) = \bigcap \{ A \cup A^* \}$
- Robot programming: formal compositions of hybrid basins
  - hybrid dynamics: make/break contacts; stabilize underactuated DoF
  - compositions: parallel; sequential; hierarchical
  - formal: correct-by-design ⇔ type theory ⇔ category

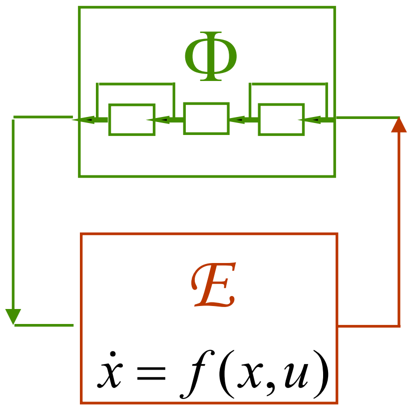


*Compositions: reuse and recombine basins*

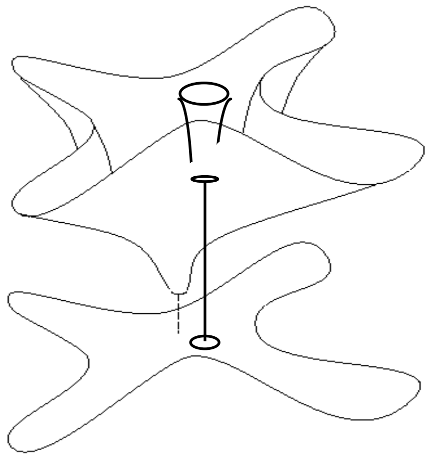



*Hybrid Basins: Conley’s Fundamental Thm.*

Physical “Behav




(instanc



$\dot{x} = f_\Phi(x)$

Dynamical “Letter,” Φ

(model)

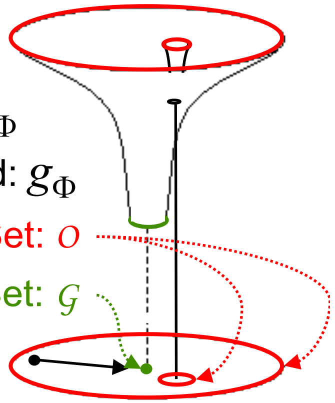
$x = h(y)$

Lyapunov Function: $\gamma_\Phi$

Vector Field: $g_\Phi$

Obstacle Set: $\mathcal{O}$

Goal Set: $\mathcal{G}$



$\dot{y} = g_\Phi(y)$

**Theorem (Conley's decomposition theorem for MHS)**

Let $H = (I, F, Z, \varphi, r)$ be a deterministic MHS. Assume that $I$ is compact and that $Z$ is a trapping guard. Further suppose that, for every $x \in I$, there is an infinite or Zeno execution starting at $x$. Then the hybrid chain recurrent set $R(H)$ admits a **Conley decomposition**:

$$R(H) = \bigcap \{A \cup A^* \mid A \text{ is an attracting set for } H.\}.$$

Furthermore, $x, y \in I$ are chain equivalent if and only if either $x, y \in A$ or $x, y \in A^*$ for every attracting-repelling pair $(A, A^*)$.

**Theorem (Conley's fundamental theorem for MHS)**

Let $H = (I, F, Z, \varphi, r)$ be a deterministic MHS. Assume that $I$ is compact and that $Z$ is a trapping guard. Further suppose that, for every $x \in I$, there is an infinite or Zeno execution starting at $x$. Then there exists a **complete Lyapunov function for $H$**.

[Kvalheim et al., SIADS’21]

Penn Engineering | GRASP Laboratory
General Robotics, Automation, Sensing & Perception Lab

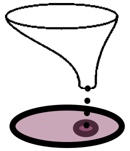
# Outline

- Bottom Up: Reactive Letters to Syllables to Words of Energy Barrier Ascent
  - Attractor Basin Compositions
  - Environment Abstraction
  - Agent Abstraction
  - Joint Level Reactive Planning
- Top Down: Reactive Global Planning in Partially Known Environments
  - Navigation Functions
  - Environment Abstraction
  - Integrating Reactive Motion Planners into Deliberative Architectures
- Toward a Physically Grounded Formal Language of Work
  - First Steps: Hybrid Dynamical Systems Category
  - On the Horizon: Hybrid Dynamical Systems Type Theory

# Outline

- Bottom Up: Reactive Letters to Syllables to Words of Energy Barrier Ascent
  - Attractor Basin Compositions
  - Environment Abstraction
  - Agent Abstraction
  - Joint Level Reactive Planning
- Top Down: Reactive Global Planning in Partially Known Environments
  - Navigation Functions
  - Environment Abstraction
  - Integrating Reactive Motion Planners into Deliberative Architectures
- Toward a Physically Grounded Formal Language of Work
  - First Steps: Hybrid Dynamical Systems Category
  - On the Horizon: Hybrid Dynamical Systems Type Theory

# Attractor Basin Compositions of Behavior

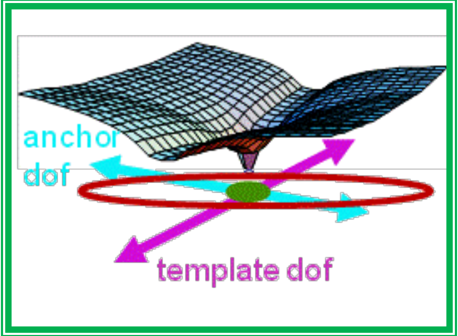


- Hierarchical ("templates & anchors") [Full & Kod, JEB'99]

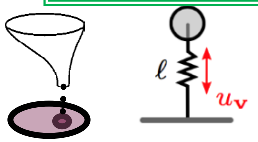
[Buehler *et al.* CSM'90]

[Schwind *et al.* ICRA'95]

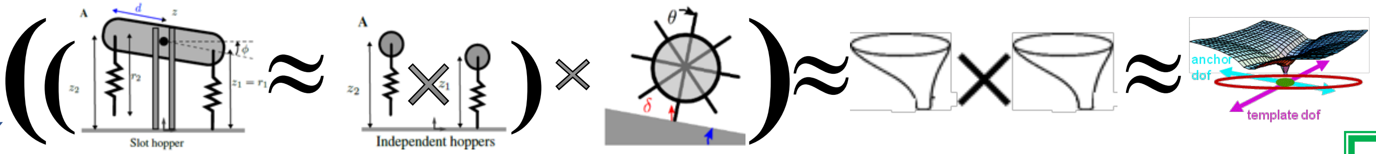
[ Saranli *et al.* IJRR'01]

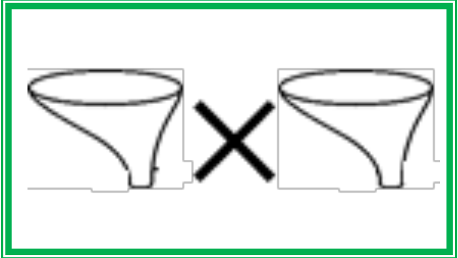



- Parallel




[ De & Kod, IJRR'18]

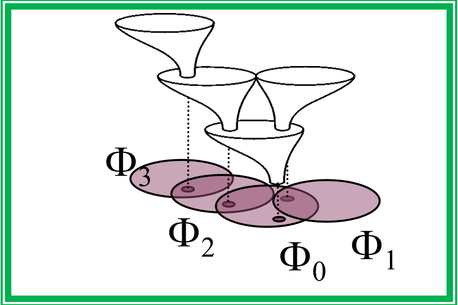


- Sequenced Transitions vs. Sequential

[Topping et al. IROS'17]

[Topping et al. ISRR'19]



# GRC: Represent the (Sagittal) Environment

[Johnson & Kod, ICRA 2013]

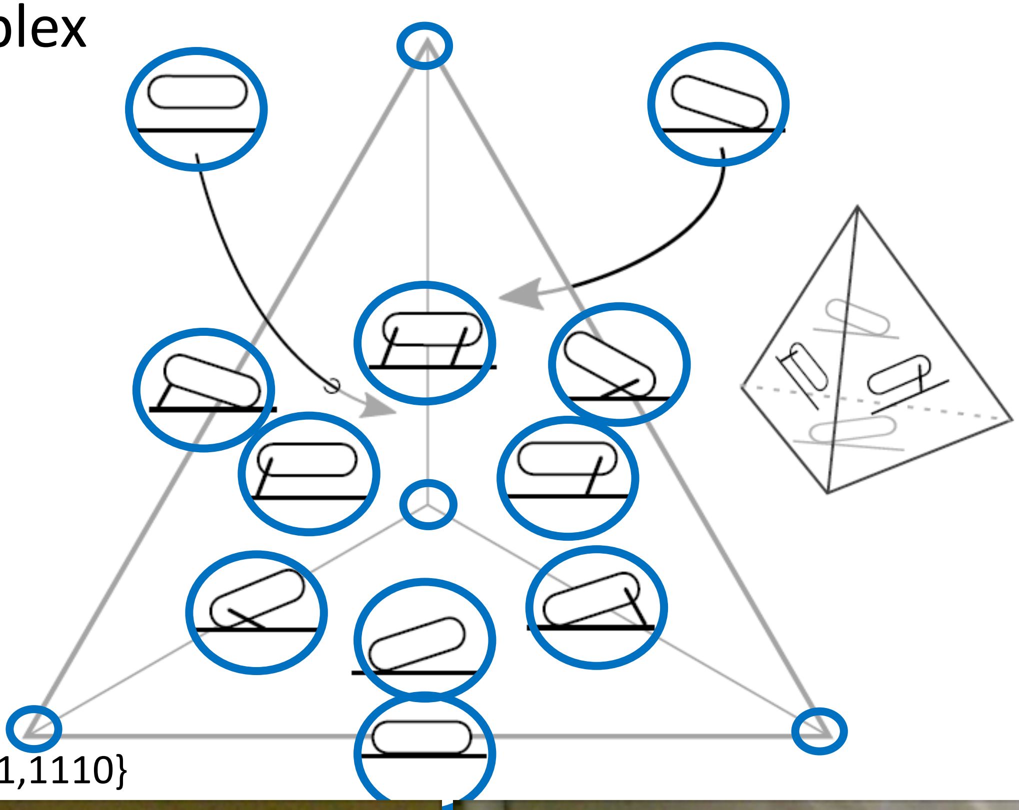
## Ground Reaction Complex

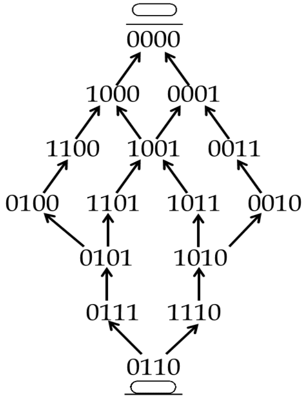
Assumptions:

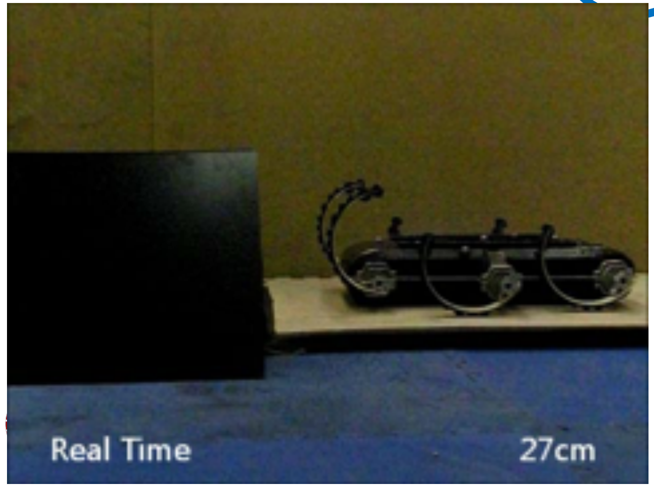
- Planar, Single Substrate
- 2 Point Slippery Body
- 2 Sticky Toes
- Massless Legs
- Single orientation

16 "Environments":

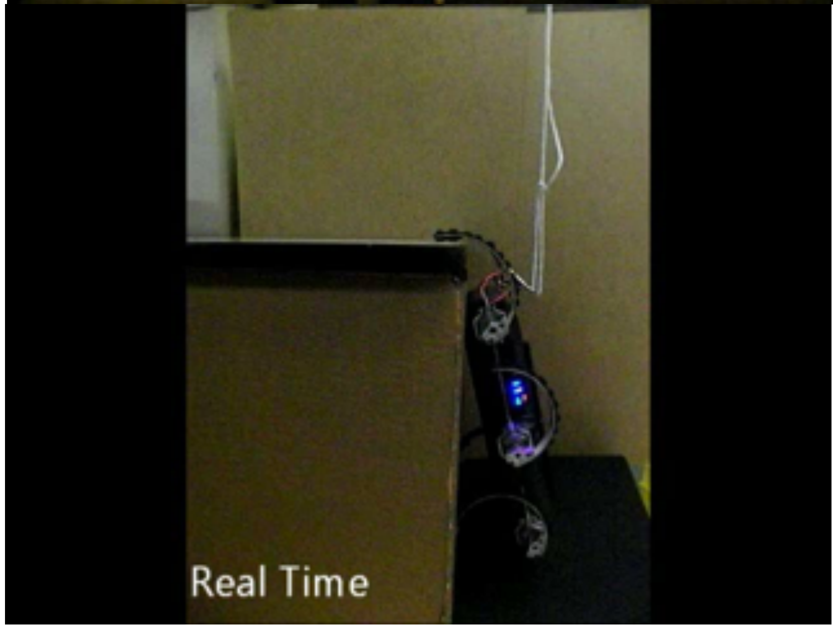
- 1 Aerial 3 DOF {0000}
- 2 Open chain 2 DOF {0100,0010}
- 2 Nose slide 2 DOF {1000,0001}
- 1 Standing 1 DOF  {0110}
- 2 Crank-slider 1 DOF {0101,1010}
- 2 Single link 1 DOF {0101,1010}
- 1 Sliding 1 DOF {1001}
- 4 Vertices 0 DOF {0111,1011,1101,1110}







# Template-Reactive Energy Barrier Ascent

- $(2n)^k$ modes ("environments") e.g.,
  - e.g., $k$=2 legged machine
  - e.g., $n$=4 annotated substrate segments
    - geometric variables
    - stick vs slip friction

  $\Rightarrow \sim O[(2n)^k!]$ different mode sequences

  $\Rightarrow$ ?? $\sim O[(2n)^k!]$ sequence controllers ?? ☹

- Compositional mode-reactive (edge-open loop) ascent
  - plan path (edge sequence) up GRC
  - closed loop composition rules for $m$=4 templates

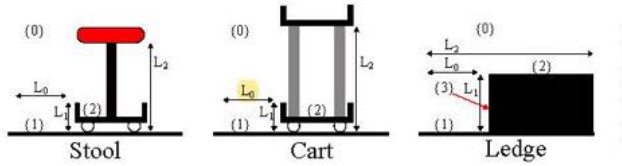
  $\Rightarrow$ 1 anchoring controller/template/mode

  $\Rightarrow \sim O[m(2n)^k]$ tuned controller compositions

  (with guard-targeted basins)

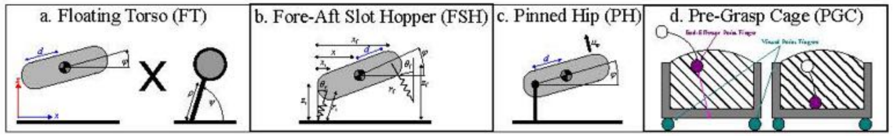
- Needed : "universal" (sagittal plane) template with automated anchoring controllers

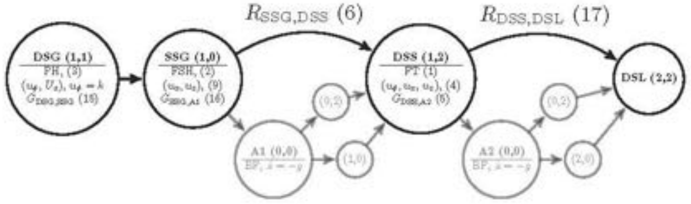
$\Rightarrow \sim O[(2n)]$ tuned controller compositions

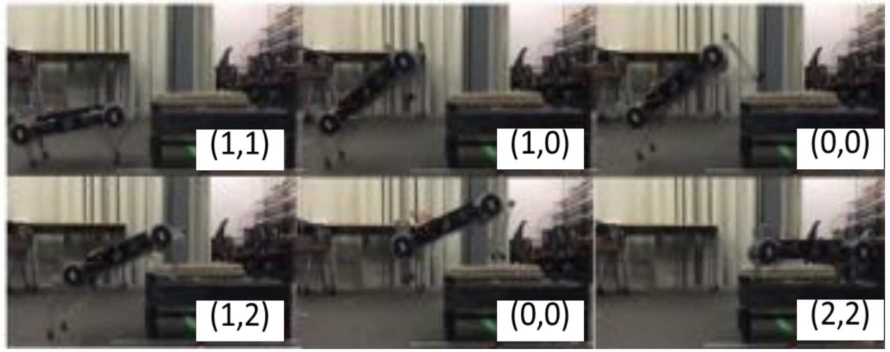


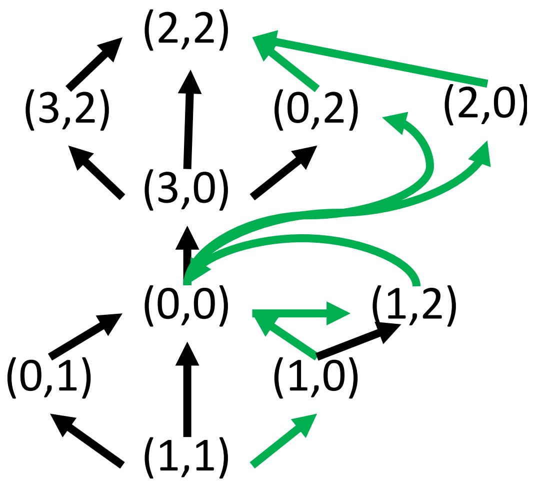

| Common Contact Modes | |
|---|---|
| Aerial (A) | (0,0) |
| Single Support, Ground (SSG) | (1,0) |
| Double Support, Ground (DSG) | (1,1) |
| Double Support, Split (DSS) | (1,2) |
| Double Support, Ledge (DSL) | (2,2) |

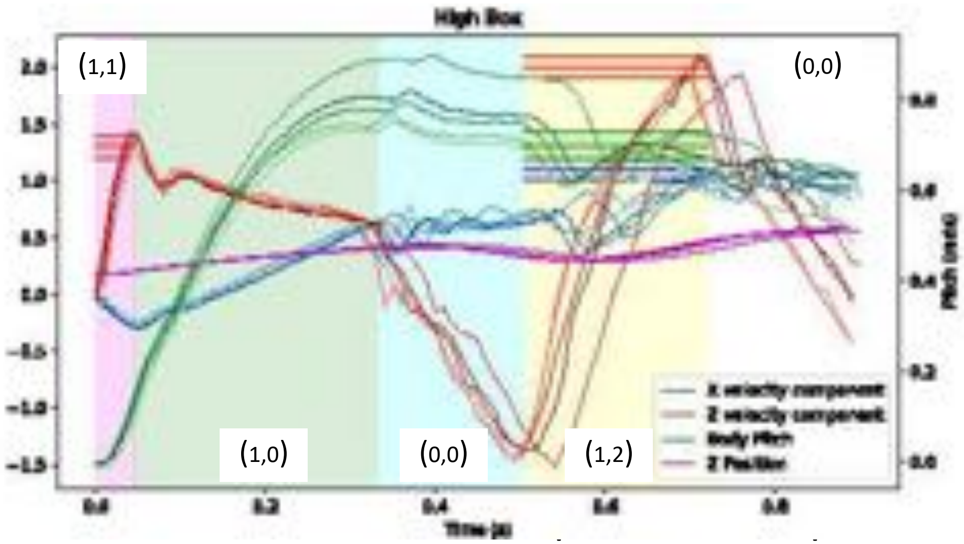
[Topping, et al., ISRR 2019]





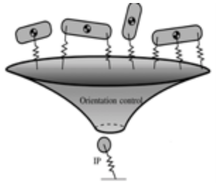
Penn Engineering | GRASP Laboratory
General Robotics, Automation, Sensing & Perception Lab

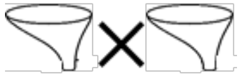
# Ascent via Anchored Template Words

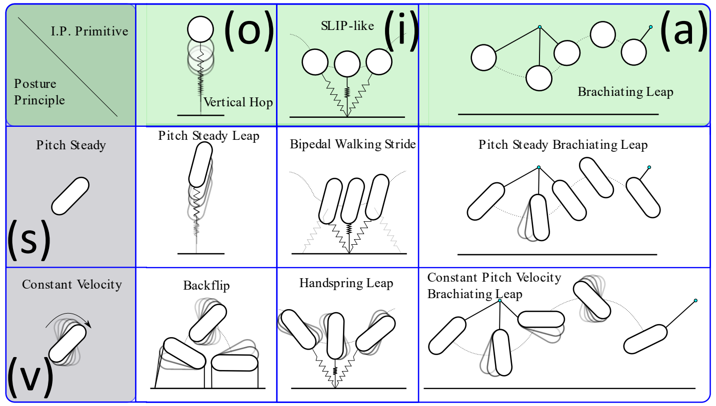
[De *et al.* Access'22]

[Topping & Kod (*in prep*)]

- Universal (sagittal plane) Template
  - anchored via parallel composition
  - in arbitrary limbed lamina
- Palette of templates
  - pendula: 2 DoF CoM
    - inverted
    - hanging
  - velocity regulated pitch
- Compositions
  - programmed in template
  - executed in anchor
  - sample behaviors
    - tunable backflips (height matched to pitching velocity)
    - brachiating leaps
    - sequential composition of brachiating & hopping leaps

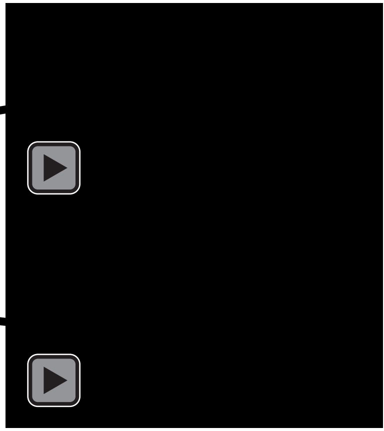


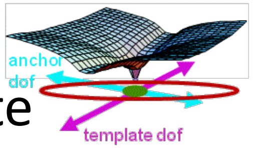



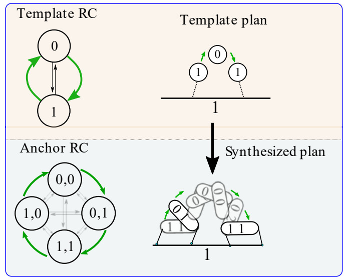



(sa)

(va)

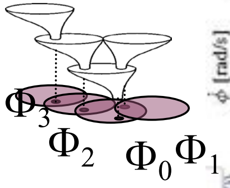


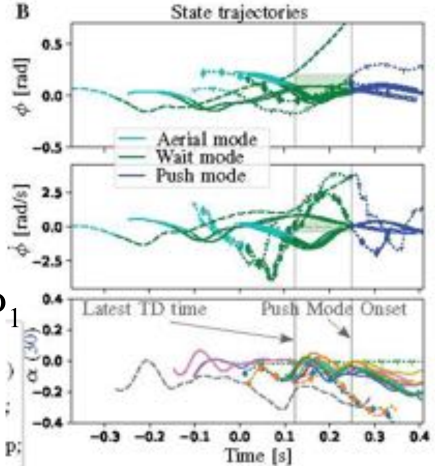



(vi) @ 270°

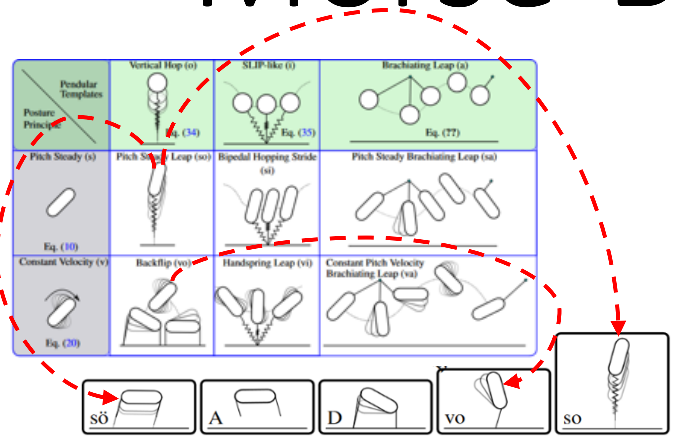
(vi) @360°

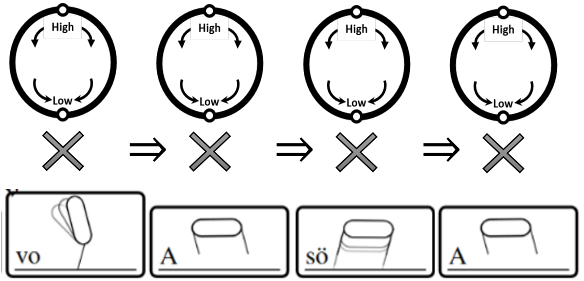


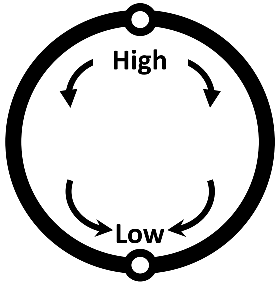
# Morse-Bott Anchors the SagittalPlane

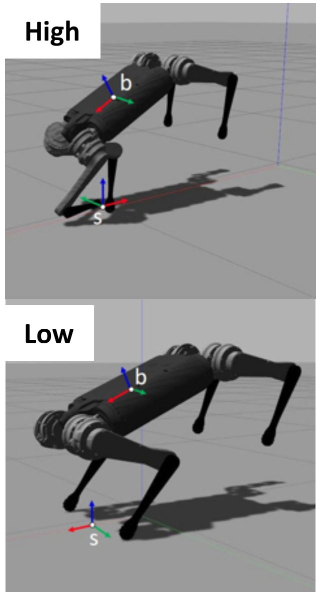
Greco & Kod (submitted)

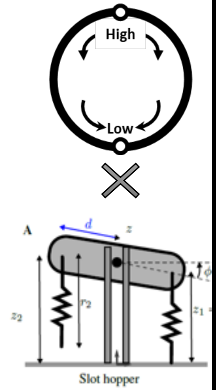



- Potential-Dissipative Control [Lord Kelvin]
  - all motion ends up at extrema
  - “almost” all ends up at minima
- Nondegenerate Smooth Potentials
  - point extrema [Morse]
  - set extrema [Bott]
- Smooth Rotation Group Potentials
  - non-empty minimum set
  - implies non-empty maximum set



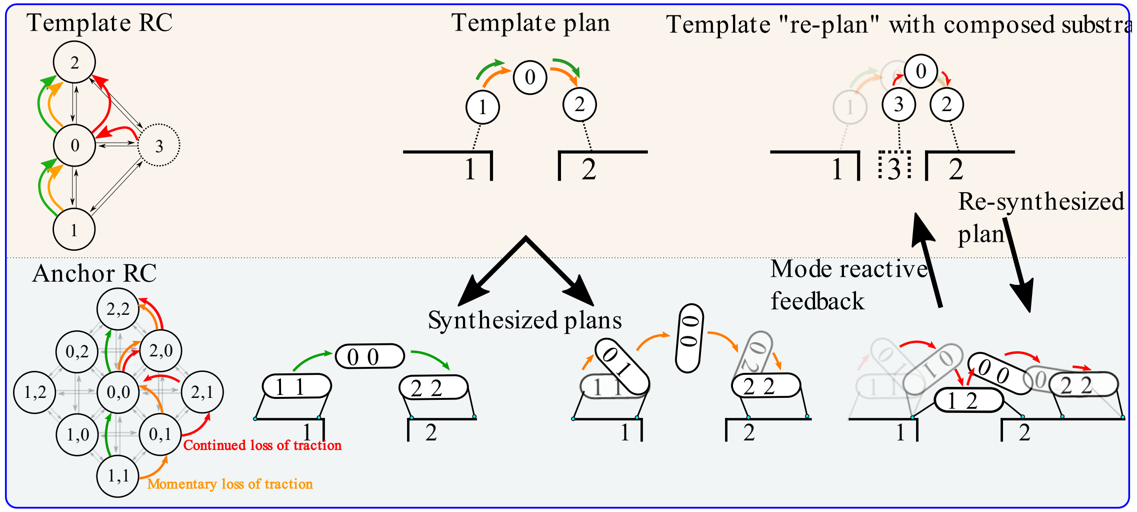




Penn Engineering | GRASP Laboratory
General Robotics, Automation, Sensing & Perception Lab

# Toward GRC-edge Reactive Planning

[Topping & Kod (*in prep*)]

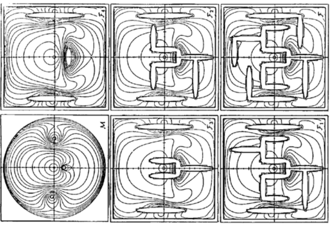
- From edge-open to edge-reactive ascent
  - edge-open: detect missed edge
    - ⇒ today’s robots give up ☹
  - edge-reactive template plan
    - replace path (edge sequence)
    - with (sequentially pruned) Hasse diagram
  - edge-reactive anchored execution
    - detect mode
    - deploy best reachable edge
    - repeat until goal or dead-end



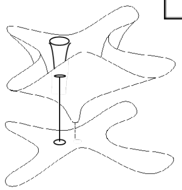

# Outline

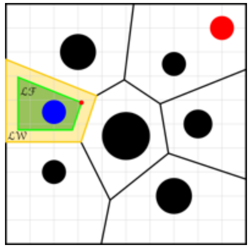
- Bottom Up: Reactive Letters to Syllables to Words of Energy Barrier Ascent
  - Attractor Basin Compositions
  - Environment Abstraction
  - Agent Abstraction
  - Joint Level Reactive Planning
- Top Down: Reactive Global Planning in Partially Known Environments
  - Navigation Functions
  - Environment Abstraction
  - Integrating Reactive Motion Planners into Deliberative Architectures
- Toward a Physically Grounded Formal Language of Work
  - First Steps: Hybrid Dynamical Systems Category
  - On the Horizon: Hybrid Dynamical Systems Type Theory

# Deformably Reactive Global Motion Planning & Execution

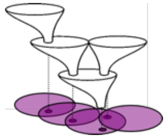


[Rimon & Kod, TAMS'92]

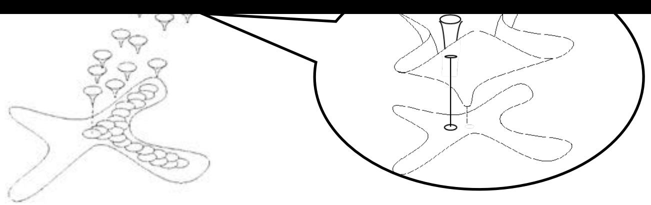




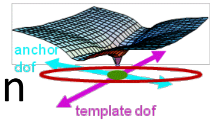
[Arslan & Kod, IJRR'19]

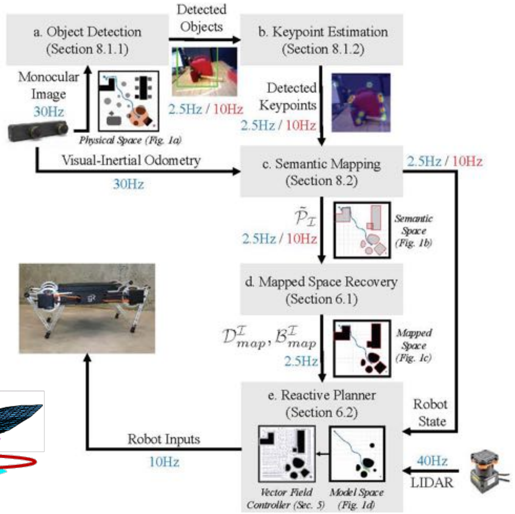


[Vasilopoulos *et al.*, IJRR'22]

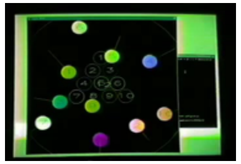


- Reactive Navigation
  - complete prior knowledge
  - fully perception driven
    - unknown sphere worlds
    - partially known worlds
- Emerging Architecture
  - semantic perception
    - learned obstacle classes
    - semantic SLAM
  - horizontal plane dynamics
    - planar point particle
    - differential drive
- Hierarchical composition
  - template ↪ anchor
    - fore-aft velocity ↪ pogo-stick
    - pogo-stick ↪ legged sagittal lamina
    - legged sagittal lamina ↪ spatial quadruped
  - reactive ↪ deliberative

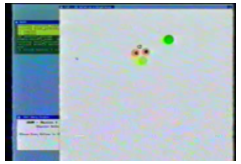



[Vasilopoulos *et al.*, RAL'20 ]



[Whitcomb & Kod, IROS'91]



[Karagoz, Bozma & Kod, TRO'04]

[Vasilopoulos *et al.*, IROS'18 ]

[Vasilopoulis *et al.*, ICRA'22]

Penn Engineering | GRASP Laboratory
General Robotics, Automation, Sensing & Perception Lab

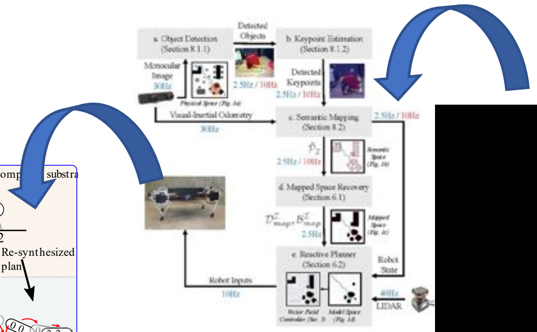
# Outline

- Bottom Up: Reactive Letters to Syllables to Words of Energy Barrier Ascent
  - Attractor Basin Compositions
  - Environment Abstraction
  - Agent Abstraction
  - Joint Level Reactive Planning
- Top Down: Reactive Global Planning in Partially Known Environments
  - Navigation Functions
  - Environment Abstraction
  - Integrating Reactive Motion Planners into Deliberative Architectures
- Toward a Physically Grounded Formal Language of Work
  - First Steps: Hybrid Dynamical Systems Category
  - On the Horizon: Hybrid Dynamical Systems Type Theory

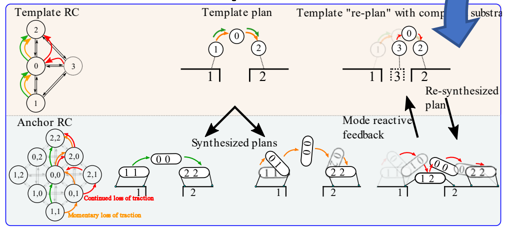
# Aim: Physically Grounded Formal Language of Work

- Physical (top down): deformably reactive planners

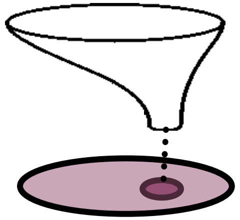
[Vasilopoulis *et al.*, ICRA'21]



- Physical (bottom up): anchor-reactive templates
  - detect mode
  - deploy best reachable edge
  - repeat until goal or dead-end




- Formal: "double category" of hybrid dynamical systems
  - basins: Conley's fundamental theorem
  - deformation: "vertical" hybrid semiconjugacies
  - sequential: "horizontal" directed systems
  - hierarchical: "pullback" hybrid subdivisions
  - parallel: still under development



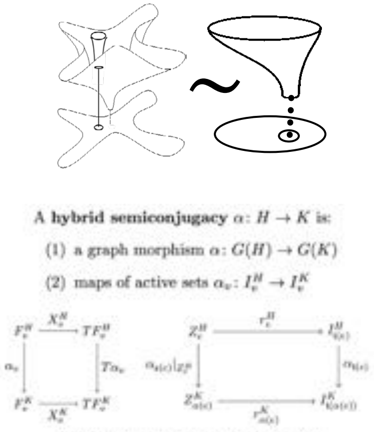
**Theorem (CGKS)**

*Hybrid systems form a double category $\mathscr{H}$ with 2-cells:*

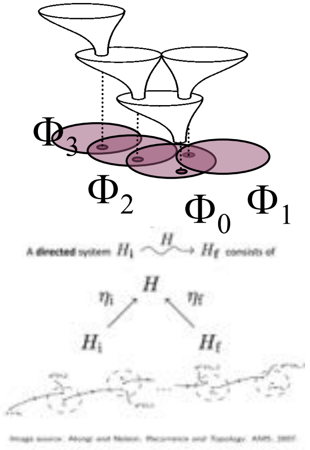
$$\begin{tikzcd} M \arrow[r, "H"] \arrow[d, "f"'] \arrow[dr, phantom, "\zeta\Downarrow"] & N \arrow[d, "g"] \\ P \arrow[r, "H'"'] & Q \end{tikzcd}$$

- **Objects:** $M, N, P, Q$ are continuous systems on Riemannian manifolds
- **Vertical arrows:** $f, g$ are smooth maps
- **Horizontal arrows:** $H, H'$ are hybrid systems
- **2-cells:** $\zeta$ is a generalized hybrid semiconjugacy

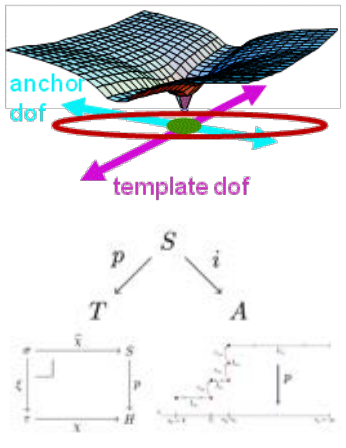
[Culbertson *et al.* TAC'20]




Hybrid semiconjugacies




Directed systems




Hybrid subdivisions

[Gustafson *et al.*(arXiv'21)]

$$\begin{aligned}
Obstacle &= (center : X) \times (radius : \mathbb{R}_{>0}) \\
SeparationViolation &= (O_i, O_j : Obstacle) \to d(O_i, O_j) \leq 2R \\
go &: (g : X, n : \mathbb{N}) \to (c : (s : See(n)) \multimap (At(g) \otimes See(n)) \oplus Interrupt(s)) \\
Interrupt &: See(n) \multimap NewObs(See(n+1)) \oplus LoseObs(See(n-1)) \\
&\quad \oplus SV((O_1, O_2 : Obstacle) \otimes SeparationViolation(o1, o2)) \\
detect &: See(n) \multimap See(n-1) \oplus See(n) \oplus See(n+1) \\
visibleObs &: !See(n) \to List(Obstacle) \\
projGoal &: ConvHull(n) \to X \to X \\
voronoi &: !See(n) \to ConvHull \\
ConvHull &= List(X) \\
startSensing &: Unit \multimap (n : \mathbb{N}) \otimes See(n) \\
stopSensing &: (n : \mathbb{N}) \to See(n) \multimap Unit \\
controller &: (g : X) \to d(x, nearestObs(s)) > R) \\
&\quad \to (f : Free \multimap At(g) \oplus (O_i, O_j : Obstacle) \otimes SeparationViolation(O_i, O_j)) \otimes Safe(f
\end{aligned}$$

# Bibliography


- [Arslan & Kod, IJRR'19] Arslan, Omur, and Daniel E Koditschek. "Sensor-Based Reactive Navigation in Unknown Convex Sphere Worlds." *The International Journal of Robotics Research* 38, no. 2–3 (March 1, 2019): 196–223. https://doi.org/10.1177/0278364918796267.
- [Buehler et al, CSM'90] Buehler, M., D. E. Koditschek, and P. J. Kindlmann. "A Family of Robot Control Strategies for Intermittent Dynamical Environments." *IEEE Control Systems Magazine* 10 (1990): 16–22.
- [Burridge et al., IJRR'99] Burridge, R. R., A. A. Rizzi, and D. E. Koditschek. "Sequential Composition of Dynamically Dexterous Robot Behaviors." *The International Journal of Robotics Research* 18, no. 6 (June 1999): 534–55. https://doi.org/10.1177/02783649922066385.
- [Culbertson et al.,TAC'20] Culbertson, Jared, Paul Gustafson, Daniel E. Koditschek, and Peter F. Stiller. "Formal Composition of Hybrid Systems." *Theory and Applications of Categories* 35, no. 45 (October 9, 2020): 1634–82.
- [De et al., Access'22] A. De, T. T. Topping, J. D. Caporale, and D. E. Koditschek, "Mode-Reactive Template-Based Control in Planar Legged Robots," *IEEE Access*, vol. 10, pp. 16010–16027, 2022, doi: 10.1109/ACCESS.2022.3148921.
- [De & Kod, IJRR'18] De, Avik, and Daniel E. Koditschek. "Vertical Hopper Compositions for Preflexive and Feedback-Stabilized Quadrupedal Bounding, Pacing, Pronking, and Trotting." *The International Journal of Robotics Research* 37, no. 7 (June 1, 2018): 743–78. https://doi.org/10.1177/0278364918779874.
- [Full & Kod, JEB'99] Full, Robert, and Daniel Koditschek. "Templates and Anchors: Neuromechanical Hypotheses of Legged Locomotion on Land." *J. of Experimental Biology* 202, no. 23 (1999): 3325–32.
- [Gustafson et al. arXiv'21] P. Gustafson, J. Culbertson, and D. E. Koditschek, "Hybrid dynamical type theories for navigation," *arXiv:2108.07625 [cs, math]*, Aug. 2021, Accessed: Oct. 24, 2021. [Online]. Available: http://arxiv.org/abs/2108.07625
- [Johnson & Kod, ICRA'13] Johnson, Aaron M., and D. E. Koditschek. "Toward a Vocabulary of Legged Leaping." In *Robotics and Automation (ICRA), 2013 IEEE International Conference On*, 2568–75. IEEE, 2013. https://doi.org/10.1109/ICRA.2013.6630928.
- [Karagoz et al, TRO'04] Karagoz, C. S., H. I. Bozma, and D. E. Koditschek. "Feedback-Based Event-Driven Parts Moving." *Robotics, IEEE Transactions on [See Also Robotics and Automation, IEEE Transactions On]* 20, no. 6 (2004): 1012–18.
- [Kod, Annu. Rev. CRAS'21] Koditschek, Daniel E. "What Is Robotics? Why Do We Need It and How Can We Get It?" *Annual Review of Control, Robotics, and Autonomous Systems* 4, no. 1 (May 2021): 1–33. https://doi.org/10.1146/annurev-control-080320-011601.
- [Kvalheim et al., SIADS'21] Kvalheim, Matthew D., Paul Gustafson, and Daniel E. Koditschek. "Conley's Fundamental Theorem for a Class of Hybrid Systems." *SIAM Journal on Applied Dynamical Systems*, January 1, 2021, 784–825. https://doi.org/10.1137/20M1336576.
- [Rimon& Kod, TAMS'92] Rimon, E., and D. E. Koditschek. "The Construction of Analytic Diffeomorphisms for Exact Robot Navigation on Star Worlds." *Transactions of the American Mathematical Society* 327, no. 1 (1991): 71–116.
- [Saranli et al, IJRR'01] Saranli, U., M. Buehler, and D. E. Koditschek. "RHex: A Simple and Highly Mobile Hexapod Robot." *The International Journal of Robotics Research* 20, no. 7 (2001): 616.
- [Schwind & Kod, ICRA'95] Schwind, William J., and Daniel E. Koditschek. "Control of Forward Velocity for a Simplified Planar Hopping Robot." In *Robotics and Automation, 1995. Proceedings., 1995 IEEE International Conference On*, 1:691–96. IEEE, 1995. http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=525364.
- [Topping et al., IROS'17] Topping, T. Turner, Gavin Kenneally, and D. E. Koditschek. "Quasi-Static and Dynamic Mismatch for Door Opening and Stair Climbing with a Legged Robot." In *Robotics and Automation (ICRA), 2017 IEEE International Conference On*, 1080–87. IEEE, 2017. https://doi.org/10.1109/ICRA.2017.7989130.
- [Topping et al, ISRR'19] Topping, T. Turner, Vasileios Vasilopoulos, Avik De, and Daniel Koditschek E. "Composition of Templates for Transitional Pedipulation Behaviors." In *Proc. Int. Symp. Rob. Res.*, https://repository.upenn.edu/ese_papers/860/, 2019. https://repository.upenn.edu/ese_papers/860/.
- [Vasilopoulos et al, IROS'18] Vasilopoulos, V., T. T. Topping, W. Vega-Brown, N. Roy, and D. E. Koditschek. "Sensor-Based Reactive Execution of Symbolic Rearrangement Plans by a Legged Mobile Manipulator." In *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 3298–3305, 2018. https://doi.org/10.1109/IROS.2018.8594342.
- [Vasilopoulos *et al.*, RAL'20 ] Vasilopoulos, Vasileios, Georgios Pavlakos, Sean L. Bowman, J. Diego Caporale, Kostas Daniilidis, George J. Pappas, and Daniel E. Koditschek. "Reactive Semantic Planning in Unexplored Semantic Environments Using Deep Perceptual Feedback." *IEEE Robotics and Automation Letters* 5, no. 3 (July 2020): 4455–62. https://doi.org/10.1109/LRA.2020.3001496.
- [ Vasilopoulos et al., ICRA'22] V. Vasilopoulos, S. Castro, W. Vega-Brown, D. E. Koditschck, and N. Roy, "A Hierarchical Deliberative-Reactive System Architecture for Task and Motion Planning in Partially Known Environments," in *2022 International Conference on Robotics and Automation (ICRA)*, May 2022, pp. 7342–7348. doi: 10.1109/ICRA46639.2022.9811936.
- [Vasilopoulos et al., IJRR'22] V. Vasilopoulos, G. Pavlakos, K. Schmeckpeper, K. Daniilidis, and D. E. Koditschek, "Reactive navigation in partially familiar planar environments using semantic perceptual feedback," *The International Journal of Robotics Research*, vol. 41, no. 1, pp. 85–126, Jan. 2022, doi: 10.1177/02783649211048931.

# Working Compositions for Correct Execution of Robot Task Specifications

*D. E. Koditschek*

*ESE Department, University of Pennsylvania*

*Philadelphia, PA 19104*

***The research discussed in this talk has been supported by***

OSD/R&E(BRO)/ONR: Vannevar Bush Fellowship (N00014-16-1-2817)
AFOSR: FA9550-19-1-0265; AFRL: FA865015D1845 sub 669737-1D
ARO: W911NF-17-1-0229; ARO: SLICE MURI W911NF1810327
NSF: LLDR (INSPIRE-1514882); CoRoGeoFR (NRI-II-1734355)